Grade: 58/60 = 97%

Graded by Lawrence Jerome, 7/28/2004.

An excellent start in Discrete Math, just two partial mistakes! I've made my comments in blue, and points subtracted in red.

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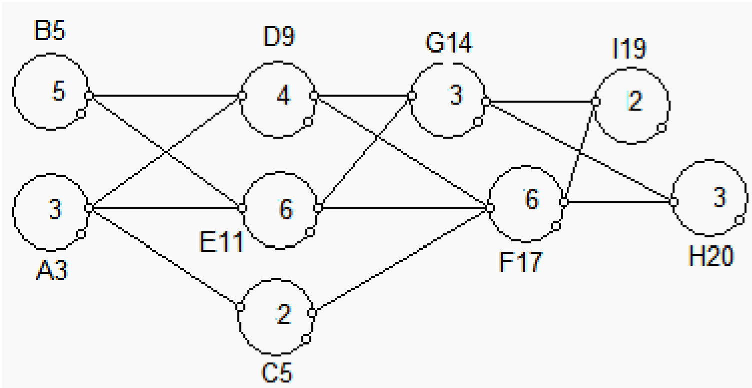
1.1 The table below tells the time needed for a number of tasks and which tasks precede them. Make a PERT diagram, and determine the project time and critical path.

Task	Time	Preceding Tasks
A	3	NONE
B	5	NONE
C	2	A
D	4	A, B
E	6	A, B
F	6	C, D, E
G	3	D, E
H	3	F, G
I	2	F, G

The project time is twenty. The critical path is B-E-F-H. [Correct.]



1.2 Calculate:

a. 8!/5!

$$\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1}=8\cdot7\cdot6=336$$ [Correct.]

b. 9!/(3! 6!)

$$\frac{9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}=3\cdot4\cdot7=84$$ [Correct.]

1.3 Let A = {1, 2}, B = {2, 3, 4}, C = {2}, D = {x: x is an odd positive integer}, and E = {3, 4}. Are each of the following true or false?

a. $C \subseteq A$ True

b. $B \subseteq D$ False

c. |B| = 3 True

d. $\varnothing \subseteq C$ True [Correct, all 4 parts. Good notation.]

1.4 In Cincinnati, chili consists of spaghetti topped by any (or none) of meat sauce, cheese, chopped onions, and beans. In how many ways can chili be ordered?

This allows for sixteen different ways the chili can be served. One for no toppings, four ways for one tipping, six ways for two toppings, four ways for three toppings, and one way for four toppings to be served. [Correct. Good analysis.]

1.5 For each of the following expressions, state whether or not it is a polynomial in x, and if so give its degree.

a. $2x + 3x^{1/2} + 4$

No, but it could be looked at as one if you used u substitution. In that case it would be $2u^2 + 3u + 4$. But since, it isn't, then it is not a polynomial in x by the definition given in the book because of the exponent of one half. [Correct. Good analysis.]

b. $2^x + 3x$

As long as x is a positive integral value, it is, but since there is no way of knowing what x is, then this is also not a polynomial. [Correct. 2^x is an exponential function.]

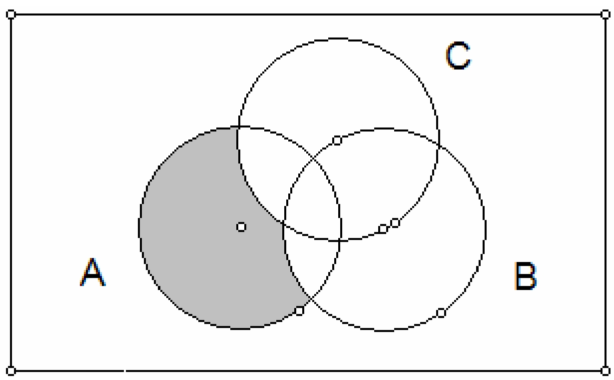
2.1 Let A = {1, 2, 3, 4}, B = {1, 4, 5}, C = {3, 5, 6}, and the universal set U = {1, 2, 3, 4, 5, 6}.

a. Determine the resulting set: $A \cap \left(\overline{B} \cup C\right)$

$\overline{B} \cup C = \{2,3,5,6\}$ so, $A \cap \left(\overline{B} \cup C\right)$ = $\{2,3\}$ [Correct.]

b. Determine the resulting set: $\overline{A} \cup \overline{C}$ = $\{1,2,4,5,6\}$ [Correct.]

c. Draw a Venn diagram depicting the set: $A - \left(B \cup C\right)$ [Correct. Good Venn diagram.]



2.2 Determine which of the reflexive, symmetric, and transitive properties are satisfied by the given relation R defined on set S, and state whether R is an equivalence relation on S.

a. S = {1,2,3,4,5,6,7,8} and x R y means that x - y is odd

The relation R is symmetric, but R is not an equivalence relation on S because it is not reflexive or transitive. [Correct, Symmetric only.]

b. S = {1,2,3,4,5,6,7,8} and x R y means that |4 - x| = |4 - y|

The relation R is reflexive and symmetric, but R is not an equivalence relation on S because it is not transitive. [Partially correct. R is also transitive and thus an equivalence relation. Partial deduction = 1 point.]

2.3 Perform the indicated operation in Z_m. Write your answer in the form [r] with $0 \le r \le m$

a. [43] + [31] in Z_{22}

$\left[43+31\right]=\left[74\right]=\left[8\right]$ [Correct.]

b. $[11]^8$ in Z_5

$\left[11\right]^8=\left[1\right]^8=\left[1^8\right]=\left[1\right]=\left[1\right]$ [Correct.]

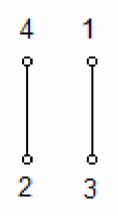
2.4 A hospital heart monitoring device uses two feet of paper per hour. If it is attached to a patient at 8 a.m. with a supply of paper 150 feet long, at what hour of the day will the device run out of paper?

It would take 75 hours to use up the tape. There are 24 hours in a day. 3 times 24 is 72, so the tape would run out three days and three hours later. This would be 11 a.m. three days later. So, if it was installed on Monday at 8 a.m., then it would run out on Thursday at 11 a.m. [Correct. Good analysis.]

2.5 For the following, determine a Hasse diagram for the partial order R on set S.

S = {1, 2, 3, 4} and R = {(1, 1), (2, 2), (3, 3), (4, 4), (2, 4), (3, 1)}

[Correct. Excellent Hasse diagram]



2.6 Identify the minimal and maximal elements of S with respect to the given partial order R.

S = the nonempty subsets of {1, 2, 3} and A R B if B $\subseteq$ A

The minimal element of S with respect to the given partial order R is {1, 2, 3}. [Correct.]

The maximal elements of S with respect to the given partial order R are {1}, {2}, {3}. [Correct. Excellent job.]

2.7 Find the value of f(a).

a. $f(x)=5x-7, a=6$ $\quad f(6)=5(6)-7=30-7=23$

b. $f(x)=\sqrt{x}-5, a=9$ $\quad f(9)=\sqrt{9}-5=3-5=-2$

c. $f(x)=\frac{4}{x}, a=\frac{1}{2}$ $\quad f\left(\frac{1}{2}\right)=\frac{4}{\frac{1}{2}}=4\cdot 2=8$

d. $f(x)=-x^2, a=-6$ $\quad f(-6)=-(-6)^2=-36$ [Correct. All calculations.]

2.8 In the following exercise Z denotes the set of integers. Determine if each function g is one-to-one, onto, or both.

a. $g: Z \to Z, g(x)=x-2$

In order to show g(x) is one-to-one, I must show that if $g(x_1)=g(x_2)$, then $x_1=x_2$. Let $g(x_1)=g(x_2)$. Then

$x_1-2=x_2-2$

$x_1=x_2$

Hence, g is one-to-one.

In order to show that g is onto, I must show that if y is an element of the codomain of g, then there is an element x of the domain such that y = g(x). Since the domain and codomain of g are both the set of real numbers, I need to show that for any real number y, there is a real number x such that y = g(x). y = x – 2, so x = y + 2. Then

$g(x)=g(y+2)$

$g(x)=(y+2)-2$

$g(x)=y$

Thus g is onto as well as one-to-one, so it is a one-to-one correspondence. [Correct. one-to-one and onto.]

b. $g: Z \to Z, g(x)=\begin{cases} x-1, & if \quad x>0 \\ x, & if \quad x\le 0 \end{cases}$

In order to show g(x) is one-to-one, I must show that if $g(x_1)=g(x_2)$, then $x_1=x_2$. Let $g(x_1)=g(x_2)$. Then

$$x_1-1=x_2-1, \quad if \quad x_1>0, x_2>0$$

$$x_1=x_2, \quad if \quad x_1>0, x_2>0$$

$$x_1=x_2, \quad if \quad x_1\le 0, x_2\le 0$$

Hence, g is one-to-one. [Incorrect. When x = 1 and x = 0, g(x) = 0, hence it is not one-to-one.

In order to show that g is onto, I must show that if y is an element of the codomain of g, then there is an element x of the domain such that y = g(x). Since the domain and codomain of g are both the set of real numbers, I need to show that for any real number y, there is a real number x such that y = g(x). y = x – 1, if x > 0, so x = y + 1. y = x, if x ≤ 0. Then

$$g(x)=g(y+1), \quad if \quad y>0$$

$$g(x)=(y+1)-1, \quad if \quad y>0$$

$$g(x)=y, \quad if \quad y>0$$

$$g(x)=y, \quad if \quad y\le 0$$

Thus g is onto as well as one-to-one, so it is a one-to-one correspondence. [Yes, g(x) is onto. Partial deduction = 1 point.]

2.9 Suppose that a number x_n is computed recursively by x_1 = 2 and $x_n = 2x_{n-1} + 3$ for n ≥ 2. Compute x_1 through x_5.

$$x_1=2$$

$$x_2=2(2)+3=4+3=7$$

$$x_3=2(7)+3=14+3=17$$

[Correct, all calculations.]

$$x_4=2(17)+3=34+3=37$$

$$x_5=2(37)+3=74+3=77$$

2.10 Prove the given statement by mathematical induction.

$$1+4+9+...+n^2=\frac{n(n+1)(2n+1)}{6}$$

$$S(n)=1+4+9+...+n^2=\frac{n(n+1)(2n+1)}{6}$$

Since S(n) is to be proved for all positive integers n, I will take the base of the induction to be $n_0=1$.

For n = 1, the left side of S(n) is 1 and the right side is

$\frac{1(1+1)(2(1)+1)}{6}=\frac{1(2)(3)}{6}=\frac{6}{6}=1$. Hence S(1) is true.

To perform the inductive step, I assume that S(k) is true for some positive integer k and show that S(k + 1) is also true.

$$S(k)=1+4+9+...+k^2=\frac{k(k+1)(2k+1)}{6}$$

To prove that S(k + 1) is true, I must show that

$$1+4+9+...+k^2+(k+1)^2=\frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

By using S(k), I an evaluate the left side of the equation to be proved as the following.

$$1+4+9+...+k^2+(k+1)^2=\frac{k(k+1)(2k+1)}{6}+(k+1)^2$$

$$=\left(\frac{2k^3+3k^2+k}{6}\right)+(k^2+2k+1)$$

$$=\left(\frac{1}{3}k^3+\frac{1}{2}k^2+\frac{1}{6}k\right)+(k^2+2k+1)$$

$$=\frac{1}{3}k^3+\frac{3}{2}k^2+\frac{13}{6}k+1$$

$$=\frac{1}{6}(2k^3+9k^2+13k+6)$$

The right hand side of the equation is proved like this:

$$\frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}=\frac{(k+1)(k+2)(2k+3)}{6}$$

$$=\frac{(k^2+3k+2)(2k+3)}{6}$$

$$=\frac{2k^3+9k^2+13k+6}{6}$$

$$=\frac{1}{6}\left(2k^3+9k^2+13k+6\right)$$

Because the left and right sides are equal in the equation to be proved, S(k + 1) is true. [Correct. Excellent job!]

Since both parts on and parts two are true, the principles of mathematical induction guarantees that S(n) is true for all integers n≥1, that is, for all positive integers n.

2.11 Evaluate the numbers.

a. C(8, 3) = $\frac{8!}{3!(8-3)!}=\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot5\cdot4\cdot3\cdot2\cdot1}=8\cdot7=56$ [Correct.]

b. C(13, 9) =

$$\frac{13!}{9!(13-9)!}=\frac{13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1\cdot4\cdot3\cdot2\cdot1}=13\cdot11\cdot5=715$$

[Correct.]

c. C(n, 1) = $\frac{n!}{1!(n-1)!}=n$ [Correct.]

d. P(n, r) / C(n, r) = $\frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}}=\frac{n!}{(n-r)!}\cdot\frac{r!(n-r)!}{n!}=r!$ [Correct.]